Beyond the minimal composite Higgs model

JHEP04(2009)070

(http://iopscience.iop.org/1126-6708/2009/04/070)

Published by IOP Publishing for SISSA

Received: *February 23, 2009*
Accepted: *March 29, 2009*
Published: *April 17, 2009*

Beyond the minimal composite Higgs model

Ben Gripaios,[a] Alex Pomarol,[b] Francesco Riva[a,c] and Javi Serra[b]

[a] *CERN PH-TH, Geneva 23, 1211 Switzerland*
[b] *Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*
[c] *Département de Physique Théorique, Université de Genève, 24, Quai Ernest Ansermet, Geneva 4, 1211 Switzerland*
E-mail: gripaios@cern.ch, alex.pomarol@uab.cat, francesco.riva@cern.ch, jserra@ifae.es

Abstract: The Higgs spectrum of the minimal composite Higgs model, based on the SO(5)/SO(4) coset, consists of a unique Higgs doublet whose phenomenology does not differ greatly from the Standard Model (SM). Nevertheless, extensions beyond this minimal coset structure exhibit a richer Higgs spectrum and therefore very different Higgs physics. We explore one of these extensions, the SO(6)/SO(5) model, whose Higgs spectrum contains a CP-odd singlet scalar, η, in addition to the Higgs doublet. Due to the pseudo-Nambu-Goldstone nature of these Higgs bosons, their physical properties can be derived from symmetry considerations alone. We find that the mass of η can be naturally light, opening up the possibility that the SM Higgs decays predominantly to the singlet, and therefore lowering the LEP bound on its mass to 86 GeV. We also show that η can have interesting consequences in flavour-violating processes, as well as induce spontaneous CP-violation in the Higgs sector. The model can also have anomalies, giving rise to interactions between the SM gauge bosons and η which, if measured at the LHC, would give quantitative information about the structure of the high energy theory.

Keywords: Higgs Physics, Beyond Standard Model, Technicolor and Composite Models

ArXiv ePrint: 0902.1483

doi:10.1088/1126-6708/2009/04/070

JHEP04(2009)070

Contents

1 Introduction

Models of electroweak symmetry breaking (EWSB) with composite Higgs bosons [1–3] have been recently reconsidered [4–10], following the stimulus of the AdS/CFT correspondence. In such models, the electroweak scale, $\Lambda_S \sim$ TeV, arises via strong-coupling effects (just as in QCD the GeV scale arises from the QCD coupling becoming strong), while the Higgs scalars appear as pseudo-Nambu-Goldstone bosons (PNGBs) of an approximate symmetry that is non-linearly realized at the electroweak scale.

The presence of strong coupling means that we are powerless to compute in general (at least in situations where the crutch of AdS/CFT is unavailable). Nevertheless, the low-energy physics of the PNGB Higgs can be described by an effective lagrangian whose terms are determined by symmetry considerations, allowing us to study them without detailed knowledge of the strong sector. This situation is similar to the pions in QCD, which at low-energies can be described by the chiral lagrangian based on the symmetry breaking pattern $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_{L+R}$.

In the case of models of EWSB, we have not yet made enough observations to fully determine what the symmetry breaking structure, $G \to H$, is. The only requirements for the symmetry pattern in the strong sector are

(i) G must contain the SM gauge group,

(ii) the PNGBs parametrizing the coset G/H must contain a Higgs doublet, and

(iii) H must contain a custodial $O(4)$-symmetry to protect $\Delta\rho$ (or the T-parameter) [11] and $Z \to b\bar{b}$ [12] from sizable corrections.

JHEP04(2009)070

The minimal model fulfilling the properties (i)–(iii) is the SO(5)/SO(4) model [4, 5], whose sigma model effective lagrangian contains 4 NGBs, making up a complex Higgs $SU(2)_L$-doublet. The SO(5) symmetry is broken by couplings to SM gauge bosons and fermions, such that these NGBs become PNGBs, getting a potential at the loop-level and driving EWSB. The measured value of the S-parameter is the only nuisance, but it appears that this too can be accommodated if one is willing to accept a tuning in the model parameters at a level of no more than one part in ten [4, 5].

Given that the SO(5)/SO(4) model provides a reasonable explanation of existing data, is there any reason to explore less minimal models with an enlarged Higgs sector? One motivation is that, as stressed above, we do not yet know what the symmetry structure is. The LHC will hopefully settle this question, but in order that it may do so, we need to be able to identify the different LHC signatures of models with different symmetry structures. As we shall see, in less minimal models the phenomenology can be dramatically changed, with implications for Higgs physics, flavour physics, and CP. In particular, a new Higgs decay channel can allow the lower bound of 114 GeV on the value of the SM Higgs mass to be evaded, and can accommodate a lighter Higgs, as preferred in composite scenarios.

Another motivation is that, as we will learn in section 3, in less minimal models with a different symmetry structure, we have the possibility of non-trivial physics associated with quantum anomalies of the symmetry. Since the anomaly is non-renormalized, the coefficients of these operators are completely fixed, up to integers that measure the fermion content of the high-energy theory. If we were able to measure these integers at the LHC or a future collider, we would be able to obtain quantitative information about the ultra-violet (UV) theory, similarly to the way in which the decay $\pi^0 \to \gamma\gamma$ allowed us to extract the number of colours in QCD.

In this Article, we explore these issues in one of the simplest extensions of the minimal composite Higgs model, the model based on the coset SO(6)/SO(5).[1] The model contains $15 - 10 = 5$ NGBs, comprising a SM Higgs doublet and an electroweak singlet η. The presence of η can lead to interesting and varied implications for phenomenology that, as we will see, crucially depend on the embedding of the SM fermions into representations of the global SO(6). We will see that these embeddings can preserve the symmetry associated with shifts of the NGB η, and protect the η mass from SM loop corrections. In particular, we will present a scenario in which the gauge and the top contributions to the η mass are zero, and therefore η can be naturally light, $\lesssim 30$ GeV, getting its mass from bottom or tau loops. This opens up the possibility of decays of the SM Higgs into the singlet, invalidating the LEP bound on the Higgs mass. The dominant decay channel of η can be $b\bar{b}$, $\tau\bar{\tau}$ or $c\bar{c}$, depending on the corresponding embeddings into SO(6) of the remaining SM fermions. If the embeddings are different for different family members, we will show that the η can mediate flavour-changing neutral currents (FCNC) and have flavour-violating decays. Furthermore, the model incorporates extra sources of CP-violation, with important implications in the Higgs sector. Since the group SO(6) is isomorphic to SU(4), the model can have an anomaly, and correspondingly a Wess-Zumino-Witten (WZW) term. This

[1] This coset was previously explored in the context of little Higgs models in ref. [13], and also in ref. [14].

term generates a coupling between η and two SM gauge bosons, and could be measured by detecting the decay channel $\eta \to \gamma\gamma$.

We will also explore models based on the SO(6)/SO(4) coset containing two Higgs doublets. Nevertheless, we will show that in these models the custodial symmetry is generically broken, implying that contributions to the T-parameter are large.

The layout is as follows. In section 2 we introduce the SO(6)/SO(5) model, describing how the SM fields are coupled to the Higgs. This allows us to determine the form of the Higgs potential, and discuss the resulting phenomenology. In section 3 we provide a discussion of anomalies and the WZW term in models based on general cosets. We give a necessary condition for a WZW term to arise, and show that this is fulfilled in the SO(6)/SO(5) model. We conclude in section 4. In appendix A, we consider a similar model based on SO(6)/SO(4) and show that it generically does not preserve the custodial symmetry. Appendix B discusses C and P in the Higgs sector of the SO(6)/SO(5) model, in the presence of a WZW term.

2 The SO(6)/SO(5) composite Higgs model

In the case that the global symmetry breaking of the strong sector is $\mathrm{SO}(6) \cong \mathrm{SU}(4) \to \mathrm{SO}(5) \cong \mathrm{Sp}(4)$ the model will contain five NGBs, transforming as a **5** of SO(5), which corresponds to a $\mathbf{1} \oplus \mathbf{4} \equiv (\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{2})$ under the subgroup $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The bi-doublet can be associated to the usual SM Higgs doublet H responsible for EWSB, while the singlet, which we denote by η, corresponds to an extra pseudoscalar state. The breaking of SU(4) down to Sp(4) can be achieved by a 4×4 antisymmetric matrix

$$\Sigma_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \tag{2.1}$$

corresponding to the vacuum expectation value (VEV) of a field Σ transforming as the **6** of SU(4):

$$\Sigma \to U \Sigma U^T \,. \tag{2.2}$$

The unbroken generators T^a satisfy

$$T^a \Sigma_0 + \Sigma_0 T^{aT} = 0 \,, \tag{2.3}$$

and correspond to the generators of Sp(4), while the broken ones, $T^{\hat{a}}$, satisfy

$$T^{\hat{a}} \Sigma_0 - \Sigma_0 T^{\hat{a}T} = 0 \,. \tag{2.4}$$

Among the ten unbroken generators we identify six corresponding to the subgroup $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ as

$$T^a_L = \frac{1}{2}\begin{pmatrix} \sigma_a & 0 \\ 0 & 0 \end{pmatrix}, \; T^a_R = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_a \end{pmatrix}, \tag{2.5}$$

while the remaining four can be taken to be

$$\frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_a \\ \sigma_a & 0 \end{pmatrix} \text{ and } \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -i\mathbb{1} \\ +i\mathbb{1} & 0 \end{pmatrix}. \tag{2.6}$$

The fluctuations along the broken directions correspond to the NGBs, which parametrize the SU(4)/Sp(4) coset

$$\Sigma = e^{\frac{i}{\sqrt{2}}\Pi/f}\Sigma_0\,, \tag{2.7}$$

where

$$\Pi = \begin{pmatrix} \eta\mathbb{1} & -i(H^c\ H) \\ i(H^c\ H)^\dagger & -\eta\mathbb{1} \end{pmatrix}, \tag{2.8}$$

with $H = \begin{pmatrix} h_3 + ih_4 \\ h_1 + ih_2 \end{pmatrix}$ and $H^c = i\sigma_2 H^*$. This can be written as

$$\Sigma = \begin{pmatrix} \left(c + i\frac{\eta s}{\sqrt{\eta^2+h^2}}\right) i\sigma_2 & \frac{s}{\sqrt{\eta^2+h^2}}(-H\ H^c) \\ -\frac{s}{\sqrt{\eta^2+h^2}}(-H\ H^c)^T & \left(c - i\frac{\eta s}{\sqrt{\eta^2+h^2}}\right) i\sigma_2 \end{pmatrix}, \tag{2.9}$$

where

$$s = \sin\frac{\sqrt{\eta^2+h^2}}{\sqrt{2}f}\,, \quad c = \cos\frac{\sqrt{\eta^2+h^2}}{\sqrt{2}f}\,, \quad \text{and} \quad h = \sqrt{h_i^2}\,. \tag{2.10}$$

By a suitable $SU(2)_L$ rotation, we can eliminate 3 NGBs (they are eaten by the SM gauge bosons), and keep only the physical Higgs, h, and η. In this gauge, the kinetic term for the PNGBs is given by

$$\begin{aligned} \frac{f^2}{8}\,\mathrm{Tr}|D_\mu\Sigma|^2 &= \frac{f^2}{2}(\partial_\mu h)^2 + \frac{f^2}{2}(\partial_\mu \eta)^2 + \frac{f^2}{2}\frac{(h\partial_\mu h + \eta\partial_\mu\eta)^2}{1-h^2-\eta^2} \\ &\quad + \frac{g^2 f^2}{4}h^2\left[W^{\mu+}W^-_\mu + \frac{1}{2\cos^2\theta_W}Z^\mu Z_\mu\right], \end{aligned} \tag{2.11}$$

where we have performed the following redefinition of the PNGB fields:

$$\frac{h^2 s^2}{\eta^2+h^2} \to h^2\,, \qquad \frac{\eta^2 s^2}{\eta^2+h^2} \to \eta^2\,. \tag{2.12}$$

Field choices related by re-definitions of this type are equally valid inasmuch as the sigma-model itself is concerned [15], but, as is clear from eq. (2.11), the redefined h is the one whose VEV sets the scale of EWSB. From now on, h and η will always refer to the redefined fields.

The gauging of the SM group breaks the global symmetry [2] SU(4) down to $SU(2)_L \times U(1)_Y \times U(1)_\eta$, where $Y = T^3_R$ and $U(1)_\eta$ is generated by

$$T^\eta = \frac{1}{2\sqrt{2}}\mathrm{Diag}(1,1,-1,-1)\,. \tag{2.13}$$

Since this latter is the symmetry under which the PNGB η shifts, gauge boson loops will generate a potential for h, but not for η.

[2] In general, gauging a subgroup K of a global symmetry breaks the global symmetry down to the largest subgroup that contains K as an ideal.

JHEP04(2009)070

2.1 Couplings to SM fermions

We now consider the couplings of the strong sector to the SM fermions. As in ref. [4], we will assume that the SM fermions couple linearly to a single operator of the strong sector (or, equivalently, to a resonance of the strong sector); these mixings will be the origin of the fermion masses. For this purpose, we need to enlarge the global group of the strong sector to include the colour group $\mathrm{SU}(3)_c$, and an extra $\mathrm{U}(1)_X$, which allows us to properly embed the hypercharges, as $Y = T^3_R + X$. This extra $\mathrm{U}(1)_X$ will not be spontaneously broken, and therefore its inclusion does not affect the results of the previous section. The PNGB fields have vanishing X-charge.

Choosing the quantum numbers of the operators in the strong sector, to which the SM fermions are coupled, is equivalent to choosing an embedding for the SM fermions into representations of the global $\mathrm{SU}(4) \times \mathrm{U}(1)_X$. Since it is not possible to embed the SM fermions into complete representations, the couplings between the SM fermions and the strong sector will, in general, break the global symmetries. We will, however, demand that these couplings preserve the custodial symmetry that protects $Zb\bar{b}$ from large corrections [12]. This means that the quark doublet must be embedded in a $(\mathbf{2},\mathbf{2})_{\mathbf{2/3}}$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$. Let us now consider, in turn, the three smallest representations of SU(4), namely the **4**, the **10** and the **6**.[3]

The **4** decomposes as $(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, and therefore can be discarded since it does not contain a $(\mathbf{2},\mathbf{2})$.

The **10**, a symmetric tensor of SU(4), decomposes into $(\mathbf{2},\mathbf{2}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3})$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. We can embed the SM quark doublet, q_L, into the $(\mathbf{2},\mathbf{2})$, while the quark singlets, u_R and d_R, can go into the $(\mathbf{1},\mathbf{3})$:

$$\Psi_q = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & Q \\ Q^T & 0 \end{pmatrix}, \quad \Psi_u = \begin{pmatrix} 0 & 0 \\ 0 & U \end{pmatrix}, \quad \Psi_d = \begin{pmatrix} 0 & 0 \\ 0 & D \end{pmatrix}, \tag{2.14}$$

where

$$Q = \begin{pmatrix} 0 & q_L \end{pmatrix}, \quad U = \begin{pmatrix} 0 & u_R \\ u_R & 0 \end{pmatrix}, \quad D = \begin{pmatrix} d_R & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.15}$$

The X-charge assignments are the following: $X_q = 2/3$, which, as discussed above, guarantees that the custodial symmetry protects $Zb\bar{b}$, and $X_u = X_d = 2/3$, in order to allow a Yukawa coupling with Σ. We notice, however, that this embedding does not break the global $\mathrm{U}(1)_\eta$ symmetry of eq. (2.13), since q_L, u_R and d_R have a well-defined transformation among themselves. Indeed, under $\mathrm{U}(1)_\eta$, we find

$$\delta\Psi_i = T^\eta \Psi_i + \Psi_i T^{\eta\,T}, \tag{2.16}$$

whence

$$\delta q_L = 0, \quad \delta u_R = -\frac{1}{\sqrt{2}} u_R, \quad \delta d_R = -\frac{1}{\sqrt{2}} d_R. \tag{2.17}$$

[3]Similar considerations apply to the conjugate $\overline{\mathbf{4}}$ and $\overline{\mathbf{10}}$ representations.

That is to say, the SM fermions have well-defined charges under $\mathrm{U}(1)_\eta$. Thus, there is a remnant $\mathrm{U}(1)_\eta$ symmetry that is broken neither by gauge nor by Yukawa interactions. What is more, if this $\mathrm{U}(1)_\eta$ is assumed to be anomalous in the background of QCD, it will be a bona fide Peccei-Quinn symmetry, solving the strong CP problem. The η will correspond to the axion, and will obtain a mass of order $m_\pi f_\pi/f$ via mixing with pions. Unfortunately, we know that an electroweak-scale axion of this type has been essentially excluded, by searches for $K^+ \to \pi^+\eta$, irrespectively of its model-dependent couplings to fermions and gauge bosons [16]. We therefore discard the **10** as well.

This leaves us with the last possibility, namely embedding the SM fermions in the **6**-dimensional representation of SU(4), carried by antisymmetric 4×4 matrices (this is the vector representation of SO(6)). Under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, this representation decomposes as $(\mathbf{2},\mathbf{2}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1})$; the SM q_L must go into the bi-doublet, while u_R and d_R each go into some linear combination of the two singlets. For the up-quark sector, we have the embedding

$$\Psi_q = \frac{1}{2}\begin{pmatrix} 0 & Q \\ -Q^T & 0 \end{pmatrix}, \quad \Psi_u = \Psi_u^+ + \epsilon_u \Psi_u^-\,, \quad \Psi_u^\pm = \frac{1}{2}\begin{pmatrix} \pm U & 0 \\ 0 & U \end{pmatrix}, \tag{2.18}$$

where $Q = (0, q_L)$, $U = u_R i\sigma_2$, and the complex parameter ϵ_u defines the embedding of the u-quark into the two singlets. As in the case of the **10**, the X-charges are $X_q = +2/3 = X_u$. For the down-sector, we are forced to embed the quark doublet into a second **6**-plet, $\Psi_{q'}$, with $X_{q'} = -1/3$. This is necessary in order to generate non-zero down-type masses, since the multiplet containing the d-quark, Ψ_d, must have $X_d = -1/3$ to give the correct hypercharge to d_R. The embeddings are then given by

$$\Psi_{q'} = \frac{1}{2}\begin{pmatrix} 0 & Q' \\ -Q'^T & 0 \end{pmatrix}, \quad \Psi_d = \Psi_d^+ + \epsilon_d \Psi_d^-\,, \quad \Psi_d^\pm = \frac{1}{2}\begin{pmatrix} \pm D & 0 \\ 0 & D \end{pmatrix}, \tag{2.19}$$

where now $Q' = (q_L, 0)$ and $D = d_R i\sigma_2$. The fact that the q_L-doublet arises from a multiplet with $X = -1/3$ implies that the custodial symmetry cannot guarantee protection of the $Zb\bar{b}$ coupling. Nevertheless, this multiplet can be assumed to be coupled to the strong sector with a small coupling $\propto \sqrt{m_b}$, assuring the generation of the bottom mass without substantially affecting the $Zb\bar{b}$ coupling. From eqs. (2.18) and (2.19) we observe that in the special case $\epsilon_i = \pm 1$ $(i = u, d)$, the SM quarks have definite charges under the $\mathrm{U}(1)_\eta$ [eq. (2.16)]:

$$\delta q_L = 0\,, \quad \delta u_R = \mp\frac{1}{\sqrt{2}} u_R\,, \quad \delta d_R = \mp\frac{1}{\sqrt{2}} d_R\,. \tag{2.20}$$

Therefore we expect to find a massless η in the limit $\epsilon_i \to \pm 1$.

2.2 One-loop effective potential

At the one-loop level, a potential for the PNGBs is generated due to the SU(4)-breaking terms arising from the SM couplings to the strong sector. This potential depends on the dynamics of the strong sector, which is in general unknown. Nevertheless, symmetry considerations are powerful enough to tell us the functional form of the potential, and to determine whether h and η can or cannot get a non-zero VEV, as well as the size of their masses. To obtain the functional form of the one-loop effective potential, we proceed in two steps. First, we write the lagrangian for the SM fields obtained by integrating out the strong sector in the background of Σ. Second, we give the one-loop potential generated by integrating over the SM fields.

Using the invariance under the global $SU(4) \times U(1)_X$, we can write the effective lagrangian for the SM gauge bosons at the quadratic level, obtained by integrating out the strong sector, as

$$\mathcal{L}_{\rm g} = \frac{1}{2} P^{\mu\nu}\left(\Pi_0^B B_\mu B_\nu + \Pi_0 \,{\rm Tr}\,[A_\mu A_\nu] + \Pi_1 \,{\rm Tr}\left[(A_\mu \Sigma + \Sigma A_\mu^T)(A_\nu \Sigma + \Sigma A_\nu^T)^\dagger\right]\right), \quad (2.21)$$

where B_μ is the $U(1)_Y$ gauge field and $A_\mu = A_\mu^a T_L^a + B_\mu T_R^3$ where A_μ^a are the gauge fields of $SU(2)_L$. The lagrangian is given in momentum-space and the Π_i are momentum-dependent form factors whose values depend on the strong dynamics. In extra dimensional models these quantities can be explicitly calculated [4]. We have also defined $P^{\mu\nu} = \eta^{\mu\nu} - p^\mu p^\nu / p^2$, where p is the momentum of the gauge fields. Using eqs. (2.9) and (2.12), and the explicit expression for the $SU(2)_L$ generators, we have

$$\mathcal{L}_{\rm g} = \frac{1}{2} P^{\mu\nu}\left[\left(\Pi_0^B + \frac{\Pi_0}{2} + \Pi_1 h^2\right) B_\mu B_\nu + \left(\frac{\Pi_0}{2} + \Pi_1 h^2\right) A_\mu^a A_\nu^a - 2\Pi_1 h^2 A_\mu^3 B_\nu\right]. \quad (2.22)$$

Similarly, for the SM quarks, the most general $SU(4)\times U(1)_X$-invariant lagrangian obtained after integrating out the strong sector is given, at the quadratic order, by [4]

$$\begin{aligned}\mathcal{L}_{\rm f} = &\sum_{r=q,u,q',d}\left[\Pi_0^r \,{\rm Tr}[\bar\Psi_r \not p \Psi_r] + \Pi_1^r \,{\rm Tr}[\bar\Psi_r \Sigma] \not p \,{\rm Tr}[\Psi_r \Sigma^\dagger]\right] \\ &+ M_1^u \,{\rm Tr}[\bar\Psi_q \Sigma]\,{\rm Tr}[\Psi_u \Sigma^\dagger] + M_1^d \,{\rm Tr}[\bar\Psi_{q'} \Sigma]\,{\rm Tr}[\Psi_d \Sigma^\dagger] + h.c.\,, \end{aligned} \quad (2.23)$$

where we have

$$\begin{aligned}
{\rm Tr}[\bar\Psi_q \Sigma] \not p \,{\rm Tr}[\Psi_q \Sigma^\dagger] &= \bar u_L \not p u_L\, h^2\,, \\
{\rm Tr}[\bar\Psi_{q'} \Sigma] \not p \,{\rm Tr}[\Psi_{q'} \Sigma^\dagger] &= \bar d_L \not p d_L\, h^2\,, \\
{\rm Tr}[\bar\Psi_u \Sigma] \not p \,{\rm Tr}[\Psi_u \Sigma^\dagger] &= 4 \bar u_R \not p u_R \left|\sqrt{1-\eta^2-h^2} + i\epsilon_u \eta\right|^2\,, \\
{\rm Tr}[\bar\Psi_d \Sigma] \not p \,{\rm Tr}[\Psi_d \Sigma^\dagger] &= 4 \bar d_R \not p d_R \left|\sqrt{1-\eta^2-h^2} + i\epsilon_d \eta\right|^2\,, \\
{\rm Tr}[\bar\Psi_q \Sigma]\,{\rm Tr}[\Psi_u \Sigma^\dagger] &= 2 \bar u_L u_R\, h \left[\sqrt{1-\eta^2-h^2} + i\epsilon_u \eta\right]\,, \\
{\rm Tr}[\bar\Psi_{q'} \Sigma]\,{\rm Tr}[\Psi_d \Sigma^\dagger] &= -2 \bar d_L d_R\, h \left[\sqrt{1-\eta^2-h^2} + i\epsilon_d \eta\right]\,.
\end{aligned} \quad (2.24)$$

The last two terms of eq. (2.23) give rise to the quark masses, so we must require that at zero momentum $M_1^{u,d} \sim m_{u,d}$. This can be achieved by requiring that the SM fermion f_i couples

[4] We have used the fact that ${\rm Tr}[\bar\Psi_q \Psi_u] = {\rm Tr}[\bar\Psi_{q'} \Psi_d] = 0$ when projected to the SM field content, eqs. (2.18) and (2.19).

to the strong sector with a strength $\propto \sqrt{m_{f_i}}$ where m_{f_i} is the fermion mass.[5] This implies

$$\Pi_1^q, M_1^u \propto m_u\,, \qquad \Pi_1^{q'}, M_1^d \propto m_d\,. \tag{2.25}$$

A similar lagrangian is obtained for the SM leptons.

Now, by integrating out the SM fields, we can get the effective potential for the PNGBs. This is expected to be dominated by one-loop effects arising from the $SU(2)_L$ gauge bosons and, due to eq. (2.25), 3rd family quarks. We find

$$\begin{aligned} V(h,\eta) &= \frac{9}{2}\int \frac{d^4p}{(2\pi)^4}\log\Pi_W \\ &\quad -(2N_c)\int \frac{d^4p}{(2\pi)^4}\left[\log\Pi_{b_L} + \log\left(p^2\Pi_{t_L}\Pi_{t_R} - |\Pi_{t_Lt_R}|^2\right)\right]\,, \end{aligned} \tag{2.26}$$

where the gauge and top propagators arise respectively from eqs. (2.22) and (2.23) with $u \to t$:

$$\begin{aligned} &\Pi_W = \frac{\Pi_0}{2} + \Pi_1 h^2, \quad \Pi_{t_L} = \frac{\Pi_0^q + \Pi_0^{q'}}{2} - \Pi_1^q h^2\,, \quad \Pi_{b_L} = \frac{\Pi_0^q + \Pi_0^{q'}}{2} - \Pi_1^{q'} h^2, \\ &\Pi_{t_R} = \Pi_0^t - \Pi_1^t 4\left|\sqrt{1-\eta^2-h^2} + i\epsilon_t\eta\right|^2, \quad |\Pi_{t_Lt_R}|^2 = |M_1^t|^2 4h^2\left|\sqrt{1-\eta^2-h^2} + i\epsilon_t\eta\right|^2. \end{aligned} \tag{2.27}$$

The functions Π_1 and M_1 characterize the effects of the spontaneous SU(4)-breaking in the strong sector, and therefore must decrease for momentum p above the scale of the strong sector Λ_S. This allows for an expansion of the logarithms in the potential that leads to an approximate formula for the potential:

$$V(h,\eta) \simeq \alpha h^2 + \lambda h^4 + |\phi|^2\left[\beta + \gamma h^2 + \delta|\phi|^2\right]\,, \qquad \phi \equiv \sqrt{1-\eta^2-h^2} + i\epsilon_t\eta\,, \tag{2.28}$$

where $\alpha, \lambda, \beta, \gamma$, and δ are constants that depend on integrals over the form factors. In the limit $\epsilon_t \to \pm 1$ in which the η becomes a true NGB, we have

$$|\phi|^2 \to (1-h^2), \tag{2.29}$$

and therefore the potential eq. (2.28) becomes η-independent. In this limit, the potential for η may be sensitive to other one-loop effects, coming from the light SM fermions. This will be the case for SM fermions f_i whose embedding parameters ϵ_i take values different from ± 1. We shall explore this possibility further later on.

We will be interested in cases in which h gets a VEV and breaks the electroweak symmetry, with η either getting a VEV, or not getting a VEV. Both of these situations can occur, for suitable values of the parameters. For example, for complex values of ϵ_t, η gets always a VEV since the term $|\phi|^2$ contains a tadpole for η. Notice that, due to the re-definition eq. (2.12), the VEVs of the PNGBs must be restricted to

$$\langle h^2\rangle + \langle \eta^2\rangle \le 1\,. \tag{2.30}$$

[5]We are assuming that for a given SM fermion the left-handed and right-handed components have similar couplings to the strong sector. This guarantees that all FCNC processes from the strong sector are suppressed (for a recent analysis see refs. [17, 18]). Relaxing this assumption can lead to large FCNC effects.

For $\epsilon_i \in \mathbb{R}$ and $\langle \eta \rangle = 0$, the Higgs h can be defined as a CP-even scalar, while η is CP-odd, as can be deduced from their couplings to fermions in eq. (2.24). This assignment is consistent with the other NGB interactions, as explained in appendix B. Even if $\epsilon_i \in \mathbb{R}$, we can have $\langle \eta \rangle \neq 0$, and then CP is spontaneously broken. We must be aware, however, that the effects of a nonzero VEV for η vanish in the limit $\epsilon_i \to \pm 1$, since the η becomes a true NGB and therefore its VEV is unphysical. When $\epsilon_i \notin \mathbb{R}$, CP is explicitly broken in the η interactions to fermions eq. (2.24). In this case we always have, as we explained above, that $\langle \eta \rangle \neq 0$, and CP is in fact broken in all Higgs interactions.

2.3 Higgs phenomenology

The Higgs physics in this model strongly depends on the values of ϵ_i which, without knowledge of the underlying strong sector, must be taken as free parameters. Two important values for these parameters are

$$\begin{aligned} \epsilon_i = \pm 1 \quad &\Longrightarrow \quad \text{No potential is generated for } \eta \text{ from loops of the fermion } f_i\,, \\ \epsilon_i = 0 \quad &\Longrightarrow \quad \text{Zero } \eta f_i \bar{f}_i \text{ coupling}\,. \end{aligned} \tag{2.31}$$

In the following, we discuss different possibilities for ϵ_i and the phenomenological implications in Higgs physics. We first consider the case in which ϵ_i are family universal, and consider later the FCNC implications when this is not the case.

Heavy-η scenario. If the value of ϵ_t is different from ± 1, then we have a scenario in which η gets a potential from top-loops. In this case, we have two physical Higgs states, h and η, with masses around $100 - 200\,\text{GeV}$. If $\langle \eta \rangle = 0$ ($\epsilon_i \in \mathbb{R}$), we have, up to effects of order $\langle h^2 \rangle$, that h behaves as the SM Higgs. The η is a CP-odd state, and couples to fermions, via eq. (2.24), with a strength

$$g_{\eta f_i f_i} = m_{f_i} \frac{\epsilon_i}{\sqrt{1 - \langle h^2 \rangle}}\,. \tag{2.32}$$

An important difference between η and h is the absence of a tree-level coupling of η to WW and ZZ, cf. eq. (2.11). By measuring, at the LHC, the different products $\sigma \times BR$, where σ represent the different production rates (either through gluon, gauge-boson fusion, or top-strahlung), and BR the possible branching ratios (decays into b, τ , γ and (virtual) weak gauge bosons), we can assure the discovery of the two Higgs states. Nevertheless, even if these can be measured, the difficult task will be to disentangle this scenario from others, e.g., supersymmetric models. This could be possible if we were able to obtain a precise determination at the LHC of the different values of the products $\sigma \times BR$ that, as explained in ref. [19], suffice to establish the composite nature of the Higgs. Another option would be to try to distinguish η from, for example, the CP-odd scalar A_0 of the MSSM. The main difference among the two arises in their coupling to hZ, which is present for A_0, but absent for η. If we can establish the presence of this coupling at the LHC, either from the decay of the CP-odd state to the CP-even one (or vice versa), or from the double production of the CP-odd and CP-even states, this will definitely rule out the scenario considered here.

JHEP04(2009)070

For the case in which $\langle\eta\rangle \neq 0$, the two Higgs states mix with each other and we end up in a scenario of two Higgs states with very similar phenomenology. The important implication in this case is that CP is violated in the Higgs sector. Nevertheless, to observe this we must rely on the decay of the Higgs to WW/ZZ, if kinematically possible, or to $\tau\bar{\tau}$, whose branching fraction is very small. These are the only two decay channels that allow a full analysis of the angular distribution of the decay products and a determination of the CP-properties of the Higgs [20]. Another suggestion is to use the angular correlations of the tagging jets in vector boson fusion production of the Higgs [21].

Light-η scenario. In the limit in which all $\epsilon_i \to \pm 1$, the η mass goes to zero, and we are driven to a very different scenario for Higgs physics. The mass of η can be below $m_h/2$, implying that the Higgs h can decay to $\eta\eta$. From eq. (2.11) we find a $h\eta\eta$ coupling [6]

$$-\frac{f^2\langle h\rangle}{2}\eta^2\partial_\mu^2 h\,, \tag{2.33}$$

which leads to a Higgs partial width

$$\Gamma(h\to\eta\eta) = \frac{m_h^3 m_W^2\beta}{8\pi g^2 f^4}\,,\quad \beta = \sqrt{1-4m_\eta^2/m_h^2}\,. \tag{2.34}$$

This decay channel can dominate over the $b\bar{b}$ channel. In the limit of $m_\eta \ll m_h$, we find

$$\frac{\Gamma(h\to\eta\eta)}{\Gamma(h\to b\bar{b})} \simeq 8.5\left(\frac{m_h}{120\text{ GeV}}\right)^2\left(\frac{500\text{ GeV}}{f}\right)^4. \tag{2.35}$$

This opens up the possibility that the Higgs could in fact be somewhat lighter than the LEP SM Higgs bound of 114 GeV, since h might have escaped detection at LEP due to the non-standard decay mode $h\to\eta\eta$ [22, 23]. For example, if $m_h \gg m_\eta \gtrsim 10$ GeV, the dominant decay mode of η is $\eta\to b\bar{b}$ and the experimental lower bound on m_h from $h\to 4b$ searches is around 110 GeV. This bound can even go down to 86 GeV for 10 GeV $\gtrsim m_\eta \gtrsim 3.5$ GeV, where the dominant decay mode is $\eta\to\tau\bar{\tau}$ [24].

Although technically natural, there is, a priori, no reason to believe that all ϵ_i should be close to ±1, but not exactly ±1 (otherwise η is a PQ-axion), and therefore one might think that the light-η scenario is not very well motivated. Nevertheless, it is perhaps reasonable to consider that the values of ϵ_i for the up-type quarks, ϵ_u, are different from those of the down-type quark, ϵ_d, or even from those of the leptons, ϵ_l, and, furthermore, that one or more of these are ±1. In this case we can find natural scenarios in which η is light. For example, if we assume $\epsilon_u = \pm1$ and $\epsilon_d \neq \pm1$, we have that η receives its mass predominantly from a b_R loop, giving

$$m_\eta^2 \sim \frac{m_b\Lambda_S^3}{16\pi^2\langle h\rangle f} \simeq (30\,\text{GeV})^2\left(\frac{\Lambda_S}{2\,\text{TeV}}\right)^3\left(\frac{500\,\text{GeV}}{f}\right)\,, \tag{2.36}$$

that is light enough to allow the decay of h to two η. The η will mainly decay to $b\bar{b}$, unless $\epsilon_d = 0$. In this latter case, we have that η does not couple to $b\bar{b}$ and decays instead to $\tau\bar{\tau}$. This decay channel can also be zero if $\epsilon_l = 0$, implying that η will mostly decay to $c\bar{c}$.

[6] There is also a coupling in the potential eq. (2.28), but it vanishes as $\epsilon_i \to \pm1$.

Another possibility is to have $\epsilon_u = \epsilon_d = \pm 1$ but $\epsilon_l \neq \pm 1$. Then the mass of η comes from loops of τ (similar to eq. (2.36)), but with $m_b \to m_\tau$), leading to a slightly lighter η. In this case, it could be kinematically forbidden for η to decay into $b\bar{b}$, its principal decay mode then being into either $c\bar{c}$ or $\tau\bar{\tau}$, depending on whether $\epsilon_l = 0$ or not.

FCNC. Let us now consider the case in which the values of ϵ_i are not family symmetric. We expect FCNC effects mediated at tree-level by η, which couples linearly to $\bar{f}^i_L f^j_R$ with a strength (assuming $\langle \eta \rangle = 0$ and $\langle h \rangle \ll 1$)

$$\mathcal{M}_{ij} = m_{f_i} \sum_k U_{R\,ik}\, \epsilon_k\, U^\dagger_{R\,kj}\,, \tag{2.37}$$

where U_R is the rotation in the right-handed sector needed to diagonalize the fermion mass matrices and i, j, k runs over all fermions. Since U_R is unitary, $U_R U_R^\dagger = 1$, we have that, as expected, $\mathcal{M}$ is diagonal for universal values of ϵ_i. We will assume that U_R is of the same order as the CKM matrix V and study the implications of non-universality of ϵ_i on flavour observables.

In the down-sector, the strongest constraints on FCNC arise from $\Delta m_K/m_K$ and ε_K. At tree-level, we have that η gives a contribution to $\Delta m_K/m_K$ given by

$$\frac{\Delta m_K}{m_K} = \frac{\mathrm{Re}[\mathcal{M}_{sd}^2]}{2 m_\eta^2 f^2 m_K} \langle K | (\bar{s}_L d_R)^2 | \bar{K} \rangle\,, \tag{2.38}$$

where $\mathcal{M}_{sd} \simeq m_s \{V_{us} V_{ud} [\epsilon_s - \epsilon_d]\}$. We find $\Delta m_K/m_K \sim 10^{-15}(100\ \mathrm{GeV}/m_\eta)^2$, which is below the experimental bound, $\Delta m_K/m_K \lesssim 7 \cdot 10^{-15}$, for $m_\eta \gtrsim 40\,\mathrm{GeV}$. The bound from ε_K can increase the bound on the η mass by a factor of 10, but this depends on the phases of ϵ_i and U_R; the constraints from $\Delta m_B/m_B$ are found to be weaker. Similarly, for the up sector, non-universal values for ϵ_i lead to contributions to $\Delta m_D/m_D$. We find that these are of order 10^{-13}, and then close to the experimental value, for $m_\eta \sim 100\,\mathrm{GeV}$. Finally, in the lepton sector, η can induce contributions to, for example, $\tau \to 3\mu$, but these are very small and only reach the experimental bound $\mathrm{BR}(\tau \to 3\mu) \lesssim 2 \cdot 10^{-7}$ for η weighing a few GeV.

An interesting consequence of having non-universal values for ϵ_i is that η can have family-violating decays with a width given by

$$\Gamma(\eta \to \bar{f}_i f_j) = \frac{N_c |\mathcal{M}_{ij}|^2 m_\eta \beta^4}{8\pi f^2}\,, \quad \beta = \sqrt{1 - m_i^2/m_\eta^2}\,, \tag{2.39}$$

where $N_c = 3$ for quarks and $N_c = 1$ for leptons, and we have assumed $m_i \gg m_j$. If kinematically allowed, the decay channel $\eta \to t\bar{c}$ can be the dominant one. Indeed, we find

$$\frac{\Gamma(\eta \to t\bar{c})}{\Gamma(\eta \to b\bar{b})} \sim \frac{|\mathcal{M}_{tc}|^2}{|\mathcal{M}_{bb}|^2} \sim \frac{m_t^2 V_{ts}^2}{m_b^2} \sim 4\,. \tag{2.40}$$

For a lighter η, the decay channel $\eta \to b\bar{s}$ could dominate over the $b\bar{b}$ channel if $\epsilon_b = 0$, since in this case one finds $\Gamma(\eta \to b\bar{s})/\Gamma(\eta \to b\bar{b}) \sim |\mathcal{M}_{bs}|^2/|\mathcal{M}_{bb}|^2 \sim V_{bc}^{-2} \gg 1$.

3 Anomalies, the Wess-Zumino-Witten term and CP

Yet another interesting aspect of the phenomenology of models based on the coset SO(6)/SO(5) is that they admit a Wess-Zumino-Witten (WZW) term in their effective lagrangian. As we shall see in more detail below, such terms are interesting for at least three reasons. First and foremost, the WZW term is the low-energy manifestation of the anomaly structure of the UV theory (just as the axial anomaly of the chiral lagrangian in hadronic physics is fixed by the quark content of QCD). Since it is non-renormalized, it opens a low-energy window onto UV physics: If it is present in a strongly-coupled theory of EWSB, and if it is observable at the LHC or a future collider, it would offer a unique opportunity to learn about the UV completion of the theory that controls the weak scale. Second, the WZW term gives the leading order correction to the two-derivative sigma model lagrangian, and, third, it plays an important rôle in the context of discrete symmetries, in particular CP.

Before discussing all this in more detail, let us first discuss, in general terms, the conditions for a WZW term to be present in a model of EWSB. For a sigma model based on the coset G/H, there are non-trivial conditions for a WZW term to be present even when the group G is not gauged. The condition [25] is that a WZW term, corresponding to an anomalous rep. of G, can be included only if the anomaly, restricted to the subgroup H, is cancelled by the H anomaly of massless fermions present in the low-energy effective theory. To see why the anomaly must match in this way, we note that the sigma model has a local H symmetry, corresponding to a compensating transformation that maintains the parametrization of the coset G/H; it is local because the coset parametrization is written in terms of the spacetime-dependent NGB fields. To maintain the Ward identities, which, in particular, guarantee that NGBs are massless, H must be anomaly free.

If we wish to go further and gauge all of G, then the H-anomaly of light fermions must itself vanish [26]. If it does not, then by sandwiching together two triangle diagrams involving the light fermions and three H gauge bosons, we can generate masses for the H gauge bosons, and these cannot be cancelled by diagrams involving a WZW term and NGBs. Then the argument of the previous paragraph tells us that the H-anomaly of any WZW term must vanish.

For theories of EWSB, we do not gauge all of G, but only some subgroup K, which intersects non-trivially with the unbroken group H. As a result, the surviving massless gauge fields belong not to K or H, but rather to some smaller subgroup J that is common to both K and H. In this general case, we can only apply the logic of the previous argument to the group J.

So in summary, a necessary condition for a WZW term is that G admits anomalous representations whose anomalies vanish when construed as anomalies of the surviving linearly-realized gauge symmetry $J \subset H, K$.

Now let us consider the implications for some specific examples. A Higgsless model with coset structure $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y/\mathrm{U}(1)_Q$ satisfies the condition for a WZW term. Indeed, a suitable anomalous rep. of G is $(2, -\frac{1}{2}\sqrt{\frac{1}{2}}) \oplus (1, \sqrt{\frac{1}{2}})$. For a model with a Higgs and a WZW term, one may consider the coset $\mathrm{SU}(3) \times \mathrm{U}(1)_X/\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ of ref. [27].

JHEP04(2009)070

However, the SO(5)/SO(4) model with a Higgs and custodial symmetry does not admit a WZW term, the reason being that SO(5) does not have anomalous representations.[7]

For an example which incorporates custodial symmetry and can have a WZW term, we need to look no further than the model based on SO(6)/SO(5) that we discussed in the previous section. Since SO(6) is locally isomorphic to SU(4), it has anomalous representations. Other examples are cosets based on SO(6)/SO(4), which we discuss in appendix A, and models based on SU(4) $\times$ SU(4)/SU(4).

These results are confirmed by consideration of the effective lagrangian. The full form of the WZW term is somewhat complicated, involving an infinite series of terms in the PNGBs; derivations in the context of holographic Higgs models are given in ref. [28] and in the context of little Higgs models in ref. [29] (some phenomenological aspects of anomalies in little Higgs models were discussed in ref. [30]). Nevertheless, at leading order in $1/f$, the WZW term gives a coupling of a PNGB to two gauge bosons via the epsilon tensor. In a Higgsless model, for example, the effective lagrangian should respect the $\mathrm{U}(1)_Q$ of electromagnetism, and indeed we can couple the charge neutral PNGB that is eaten by the Z to the electromagnetic field combination $F\tilde{F}$. By contrast, the effective lagrangian in a model with a Higgs should respect the full $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$; no operator with a single PNGB and two gauge bosons is available in a theory with just a SM Higgs (like the SO(5)/SO(4) model), but once we add a singlet (for example in an SO(6)/SO(5) or $\mathrm{SU}(3)\times \mathrm{U}(1)_X/\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model), we can write down a gauge-invariant term of the form

$$\mathcal{L} \subset \frac{\eta}{16\pi^2}(n_B B_{\mu\nu}\tilde{B}^{\mu\nu} + n_W W_{a\,\mu\nu}\tilde{W}^{a\,\mu\nu} + n_G G_{A\,\mu\nu}\tilde{G}^{A\,\mu\nu}). \tag{3.1}$$

Here, $G_{A\,\mu\nu}$, $W_{a\,\mu\nu}$ and $B_{\mu\nu}$ refer to the field strengths of the $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge group, and $\tilde{B}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}B_{\rho\sigma}/2$ and similarly for the other fields. The $n_{G,W,B}$ are integers that measure the strengths of the various anomalies and are fixed by the fermion content of the UV physics. If we were able to measure these integers at the LHC, then we would gain quantitative information about the UV physics, just as measurement of the decay rate $\pi^0 \to 2\gamma$ in hadronic physics tells us that the number of colours in QCD is three. Note that for the SO(6)/SO(5) model, the WZW terms all come from an $\mathrm{SU}(4)^3$ anomaly, such that $n_G = 0$ and $n_W = n_B$. However, the SM fermions also give contributions to the terms in eq. (3.1). Indeed, we find, in the approximation $m_{f_i} \gg m_\eta$: $\delta n_B = N_c Y_i^2 \mathrm{Re}[\epsilon_i]$, $\delta n_W = N_c \mathrm{Re}[\epsilon_i]$ for weak doublets, and $\delta n_G = \mathrm{Re}[\epsilon_i]$ for quarks. Since the relevant couplings do not respect the G symmetry, the shifts in the coefficients are not restricted to integers.

But can we measure these coefficients at the LHC, or if not, at a future collider? If we cannot measure the coefficients themselves, can we even detect the presence of these terms? We might hope to be able to produce the η directly at the LHC via WW fusion and the $\eta W\tilde{W}$ vertex, or via gluon fusion if an $\eta G\tilde{G}$ vertex is present. Alternatively, and similar to the neutral pion in QCD, we could measure the coefficients by detecting the decay of η to photons. We find

$$\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\eta \to b\bar{b})} \simeq 0.007 \left|\frac{n_\gamma}{5}\right|^2 \left(\frac{m_\eta}{100\ \mathrm{GeV}}\right)^2 \left|\frac{1}{\epsilon_b}\right|^2 , \tag{3.2}$$

[7]Identical conclusions were reached from a different direction in ref. [28].

where $n_\gamma = n_B + n_W$. Although this partial width is small, it is larger, for $n_\gamma \sim 5$, than the SM decay $\Gamma(h \to \gamma\gamma)$ which has been shown to be visible at the LHC. Furthermore, the branching ratio of η to photons can be enhanced if, as we explained in the previous section, the η cannot decay to $b\bar{b}$. We must however stress that even if we are able to observe the decay channel to photons, at the LHC we can only measure the product of the cross section and the branching ratio, not the partial width. So we cannot extract the strength of the anomaly directly, without further information. A final possibility is that once we include the higher mass resonances in the effective theory, we expect that they too will couple via the anomaly (an extra-dimensional example was recently discussed in ref. [31]), giving us another potential experimental window on the couplings.

Let us lastly discuss the connection between the WZW term and discrete symmetries, in particular CP. Discrete symmetries were, of course, the very reason for the introduction of the WZW term in the chiral lagrangian of QCD, at least in Witten's incarnation thereof [32]. To recall, the leading order (two-derivative) chiral lagrangian, $\mathrm{Tr}(\partial e^{i\pi} \partial e^{-i\pi})$ is invariant under the naïve parity, $P_0 : x \to -x$, as well as the NGB parity, $P_{\mathrm{NGB}} : \pi \to -\pi$, and charge conjugation, $C : \pi \to \pi^T$. However, of the first two, only the true parity $P = P_0 P_{\mathrm{NGB}}$ is a symmetry of QCD, and the WZW term is the leading order term in the chiral lagrangian that violates P_0 and P_{NGB} individually, while respecting P. In appendix B we show that analogous arguments go through for the EWSB model based on the coset SO(6)/SO(5): The lagrangian for the gauge and Higgs self-interactions, including the WZW term, respects CP if h and η are defined to be CP-even and CP-odd respectively.

4 Outlook

We have explored a composite Higgs model based on the coset SO(6)/SO(5), with SM fermions assigned to the **6** of SO(6). Just like the minimal composite model based on SO(5)/SO(4), the model features custodial protection of the T-parameter and $Z \to b\bar{b}$, and therefore is in agreement with EWPT if a mild tuning of v/f is accepted to accommodate the S-parameter.[8] The model contains an extra singlet scalar, η, compared to the Higgs sector of the SM, which can dramatically change the phenomenology. This strongly depends on the values of ϵ_i that, as can be seen from eq. (2.31), determines the properties of η. In particular, we have presented scenarios in which the SM Higgs can predominantly decay into 2η, which in turn can dominantly decay into any one of $b\bar{b}$, $\tau\bar{\tau}$, or $c\bar{c}$. As a result, the direct bound on the SM Higgs mass coming from LEP can be invalidated, and the true bound may in fact be as low as 86 GeV. The couplings of the singlet to SM fermions can also give rise to tree-level FCNCs that are close to the experimental bounds (or even exceeding it in the case of ε_K), and induce flavour-violating decays for η. The model can also exhibit explicit or spontaneous CP violation, though it will be difficult to test experimentally. One of the most interesting phenomenological aspects of the model is the coupling of η to gauge bosons, which is induced not only by SM loops, but also can be present if the model has anomalies. Therefore the process $\eta \to \gamma\gamma$ will be of crucial importance to unravel the underlying structure of the model.

[8] In this model the contribution to S is similar to that in the minimal composite Higgs model [4].

JHEP04(2009)070

At the LHC the most prominent way to produce the η is either through the decay of the Higgs $gg \to h \to \eta\eta$, if kinematically allowed, or from gluon fusion $gg \to \eta$. Nevertheless, η can also be produced in the decay of a heavy fermionic resonance of the strong sector. Its detection is, however, difficult. The most promising decay channel is $\eta \to \gamma\gamma$, which we expect to have a partial width larger than that of the corresponding SM Higgs decay. The phenomenological prospect at the LHC and other future colliders need, however, to be fully explored.

The model presented here can also have interesting implications for astrophysics. For example, if $\epsilon_i = 0$ for all SM fermions and there are no anomalies, the singlet is stable, and hence can be a dark matter candidate. The singlet can annihilate through the $h^2\eta^2$ interactions of eqs. (2.11) and (2.28) and these determine the relic density. The resulting physics is presumably not dissimilar from that discussed in ref. [33–35]. Another interesting question is whether electroweak baryogenesis can be realized in the model. The SM fails in this regard, because the CP violation in the CKM matrix is too small and the electroweak phase transition cannot be strongly first-order given the LEP bound on the Higgs mass. In the SO(6)/SO(5) model, the presence of the singlet could cure both of these problems. First, we have shown that, for $\langle\eta\rangle \neq 0$, the model has new sources of CP violation. Secondly, the presence of the singlet, as shown in ref. [36], can result in a strongly first-order phase transition for Higgs masses above the LEP bound. All these issues deserve further analysis.

Acknowledgments

BMG thanks R. Rattazzi for discussions. FR was supported by the European Community's Research Training Network (MEST-CT-2005-020238-EUROTHEPHY) and the Swiss National Fund. The work of AP and JS was partly supported by the Research Projects CICYT-FEDER-FPA2005-02211, SGR2005-00916, UniverseNet (MRTN-CT-2006-035863), and AP2006-03102.

A Models based on SO(6)/SO(4)

In the case in which the breaking of the SU(4) is achieved by the VEV of the symmetric representation, the **10**, the global SU(4) is broken down to SO(4). In this case, however, the nine NGBs parametrizing SU(4)/SO(4) transform as a $(\mathbf{3},\mathbf{3})$ of $SO(4) \cong SU(2)_L \times SU(2)_R$, which does not contain doublets that can be associated with the SM Higgs.

Another option is to break SU(4) by the VEV of the traceless representation, the **15**, that we denote by Ω and transforms as $\Omega \to U\Omega U^\dagger$. When the VEV of Ω takes the form

$$\Omega_0 = \mathrm{Diag}(1, 1, -1, -1)\,, \tag{A.1}$$

the global SU(4) is broken down to $SU(2)_L \times SU(2)_R \times U(1) \cong SO(4) \times SO(2)$, delivering 8 NGBs, which transform as $(\mathbf{2},\mathbf{2})_{\pm 2}$ under the unbroken subgroup. This model has two Higgs doublets, which gives rise to the following problem. While a single Higgs doublet automatically guarantees that, after EWSB, the global SO(4) symmetry of the strong sector is broken down to the custodial SO(3) symmetry that protects the T-parameter from receiving large corrections, the presence of two Higgs doublets spoils this property. The reason is that the second Higgs doublet can get a VEV, breaking the custodial SO(3) symmetry down to SO(2). To see this explicitly, let us parametrize the NGBs by the traceless, hermitian matrix

$$\Omega = e^{\frac{1}{\sqrt{2}} i\Pi_\Omega/f}\Omega_0\,, \qquad \Pi_\Omega = \begin{pmatrix} 0 & \hat{H}_1 + i\hat{H}_2 \\ \hat{H}_1^\dagger - i\hat{H}_2^\dagger & 0 \end{pmatrix}, \tag{A.2}$$

where $\hat{H}_i = (H_i^c, H_i)$. By an $SU(2)_L$ rotation, we can eliminate 3 out of the 4 components of $\hat{H}_1$ and write $\hat{H}_1 = h\mathbb{1}$. For $\hat{H}_2$, we only consider the SO(3)-breaking direction $\hat{H}_2 = -ih_3\sigma_3$. For simplicity, we will take the limit $h, h_3 \ll f$, which allows us to expand eq. (A.2):

$$\Omega \simeq \begin{pmatrix} (1-\frac{h^2+h_3^2}{4f^2})\mathbb{1} - \frac{hh_3}{2f^2}\sigma_3 & -\frac{i}{f\sqrt{2}}(h\mathbb{1}+h_3\sigma_3) \\ \frac{i}{f\sqrt{2}}(h\mathbb{1}+h_3\sigma_3) & -(1-\frac{h^2+h_3^2}{4f^2})\mathbb{1} + \frac{hh_3}{2f^2}\sigma_3 \end{pmatrix}. \tag{A.3}$$

From the kinetic term of Ω we can read off the SM gauge boson masses:

$$\frac{f^2}{8}\,\mathrm{Tr}|D_\mu\Omega|^2 = \frac{g^2}{8}(h^2+h_3^2)\left[W^{\mu+}W^-_\mu + \frac{1}{2\cos^2\theta_W}\left(1-\frac{h^2h_3^2}{2f^2(h^2+h_3^2)}\right)Z^\mu Z_\mu\right] + \cdots\,, \tag{A.4}$$

which shows that if h_3 gets a VEV, the custodial symmetry is broken and $\rho \equiv m_W^2/(m_Z^2\cos^2\theta_W) \neq 1$. Now, let us choose that the SM top be embedded in a **6** of SU(4), as in eq. (2.18) (similar results are obtained for the **10** representation). This implies that the strong sector generates the operator

$$\mathrm{Tr}[\bar{\Psi}_q \not{p}\Psi_q\Omega^*] = -\bar{u}_L \not{p} u_L \frac{hh_3}{8f^2} + \cdots\,. \tag{A.5}$$

This coupling can enter in a u_L-loop and generate (after EWSB $\langle h\rangle \neq 0$) a tadpole for h_3; this forces h_3 to get a VEV, breaking the custodial symmetry. This poses a serious problem for this type of model.

Finally, we can consider the global symmetry breaking SU(4) $\to$ SO(4) achieved by the presence of the VEV of Ω — eq. (A.1) — and Σ — eq. (2.1). In this case there are 9 NGBs transforming as $(\mathbf{1},\mathbf{1})\oplus(\mathbf{2},\mathbf{2})\oplus(\mathbf{2},\mathbf{2})$. These models, however, not only suffer from the problems discussed above but can also have sizable FCNC, since the Yukawa couplings can arise from two distinct multiplets, Σ and Ω.

B *CP*-invariance

To show that *CP* is a symmetry of the sigma model representing the Higgs sector of the SO(6)/SO(5) model, we begin by asserting that the Lie algebra of SO(6) admits two automorphisms, given by

$$\begin{aligned} A_1: &\quad T^a \to T^a, &\quad T^{\hat{a}} \to -T^{\hat{a}}\,, \\ A_2: &\quad T^a \to -T^{aT}, &\quad T^{\hat{a}} \to T^{\hat{a}T}\,, \end{aligned} \tag{B.1}$$

where T^a are the generators of the unbroken SO(5) and $T^{\hat{a}}$ are the broken generators, as in eqs. (2.3) and (2.4). Recall that an automorphism is a linear transformation among the generators that preserves the algebra. That the two transformations in eq. (B.1) preserve the algebra follows from the fact that SO(6)/SO(5) is a symmetric space: There exists a basis for Lie SO(6) (such as the explicit one given after eqs. (2.3) and (2.4)) such that

$$[T^a, T^b] \sim T^c \,, \quad [T^{\hat{a}}, T^{\hat{a}}] \sim T^a \,, \quad [T^{\hat{a}}, T^a] \sim T^{\hat{b}} \,. \tag{B.2}$$

It remains only to check that A_2, which involves transposition, can be written as a linear transformation among the generators. This is easily done using the explicit representation for the generators given after eqs. (2.3) and (2.4). Note also that since our sigma model field is written as an exponential of the broken generators, $\Sigma \sim e^{i\Pi_{\hat{a}} T^{\hat{a}}}$, these automorphisms can also be thought of as the field transformations $\Pi \to -\Pi$ and $\Pi \to \Pi^T$, just as in the chiral lagrangian for QCD.

How do these two automorphisms give rise to symmetries of the sigma model lagrangian? To answer this, we note that the general G/H coset sigma model is constructed in the following way. Firstly, given a coset representative Σ for G/H, we build the Cartan form for G, $\Sigma^{-1} d\Sigma$, which is of course an element of Lie G. Projecting this onto the subspace of broken generators, $(\Sigma^{-1} d\Sigma)_{\hat{a}}$ gives a vielbein corresponding to the natural metric on G/H. The vielbein is the basic object that we use to build the sigma model. In particular, the leading two-derivative term in the sigma-model lagrangian is just the natural metric on G/H, built out of two vielbeine, and pulled back to spacetime. Similarly, the WZW term (in $d = 4$) is built out of five vielbeine. Now the automorphisms give rise to isometries of the natural G/H metric, so any terms in the sigma-model lagrangian built out of the metric will be symmetric. The WZW term is special in that it is built not out of the metric per se, but out of the vielbein. Under the automorphism A_1 in eq. (B.1), the vielbein changes sign, and the WZW term also changes sign. So A_1 is not a symmetry of the WZW term. However, when combined with the spacetime parity operation, $P_0 : \; x \to -x$, the WZW term (which features four derivatives and an epsilon tensor) is invariant.

So we have proven that for a general G/H symmetric space, with the two automorphisms in eq. (B.1) [9] the sigma model lagrangian, including the WZW term, will be invariant under two symmetries, corresponding to A_2 and $A_1 P_0$. In the SO(6)/SO(5) model of EWSB, the combination $A_1 A_2 P_0$ corresponds precisely to

$$h \to h \,, \qquad \eta \to -\eta \,. \tag{B.3}$$

This defines the CP symmetry of the Higgs sector, including the WZW term. This is, of course, in accord with the WZW contribution eq. (3.1) which couples the CP-odd η to the CP-odd combination $F\tilde{F}$.

References

[1] D.B. Kaplan and H. Georgi, SU(2) × U(1) *breaking by vacuum misalignment*, *Phys. Lett.* **B 136** (1984) 183 [SPIRES].

[9] With the caveat that A_2 be a bona fide automorphism.

[2] H. Georgi and D.B. Kaplan, *Composite Higgs and custodial* SU(2), *Phys. Lett.* **B 145** (1984) 216 [SPIRES].

[3] M.J. Dugan, H. Georgi and D.B. Kaplan, *Anatomy of a composite Higgs model*, *Nucl. Phys.* **B 254** (1985) 299 [SPIRES].

[4] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys.* **B 719** (2005) 165 [hep-ph/0412089] [SPIRES].

[5] R. Contino, L. Da Rold and A. Pomarol, *Light custodians in natural composite Higgs models*, *Phys. Rev.* **D 75** (2007) 055014 [hep-ph/0612048] [SPIRES].

[6] R. Barbieri, B. Bellazzini, V.S. Rychkov and A. Varagnolo, *The Higgs boson from an extended symmetry*, *Phys. Rev.* **D 76** (2007) 115008 [arXiv:0706.0432] [SPIRES].

[7] A.D. Medina, N.R. Shah and C.E.M. Wagner, *Gauge-Higgs unification and radiative electroweak symmetry breaking in warped extra dimensions*, *Phys. Rev.* **D 76** (2007) 095010 [arXiv:0706.1281] [SPIRES].

[8] G. Panico, E. Ponton, J. Santiago and M. Serone, *Dark matter and electroweak symmetry breaking in models with warped extra dimensions*, *Phys. Rev.* **D 77** (2008) 115012 [arXiv:0801.1645] [SPIRES].

[9] Y. Hosotani, K. Oda, T. Ohnuma and Y. Sakamura, *Dynamical electroweak symmetry breaking in* $SO(5) \times U(1)$ *gauge-Higgs unification with top and bottom quarks*, *Phys. Rev.* **D 78** (2008) 096002 [arXiv:0806.0480] [SPIRES].

[10] C. Anastasiou, E. Furlan and J. Santiago, *Realistic composite Higgs models*, arXiv:0901.2117 [SPIRES].

[11] P. Sikivie, L. Susskind, M.B. Voloshin and V.I. Zakharov, *Isospin breaking in technicolor models*, *Nucl. Phys.* **B 173** (1980) 189 [SPIRES].

[12] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *A custodial symmetry for* $Zb\bar{b}$, *Phys. Lett.* **B 641** (2006) 62 [hep-ph/0605341] [SPIRES].

[13] E. Katz, A.E. Nelson and D.G.E. Walker, *The intermediate Higgs*, *JHEP* **08** (2005) 074 [hep-ph/0504252] [SPIRES].

[14] P. Lodone, *Vector-like quarks in a composite Higgs model*, *JHEP* **12** (2008) 029 [arXiv:0806.1472] [SPIRES].

[15] S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians.* 1, *Phys. Rev.* **177** (1969) 2239 [SPIRES].

[16] H. Georgi, D.B. Kaplan and L. Randall, *Manifesting the invisible axion at low-energies*, *Phys. Lett.* **B 169** (1986) 73 [SPIRES].

[17] C. Csáki, A. Falkowski and A. Weiler, *The flavor of the composite pseudo-Goldstone Higgs*, *JHEP* **09** (2008) 008 [arXiv:0804.1954] [SPIRES].

[18] M. Blanke, A.J. Buras, B. Duling, S. Gori and A. Weiler, $\Delta F = 2$ *observables and fine-tuning in a warped extra dimension with custodial protection*, *JHEP* **03** (2009) 001 [arXiv:0809.1073] [SPIRES].

[19] G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, *The strongly-interacting light Higgs*, *JHEP* **06** (2007) 045 [hep-ph/0703164] [SPIRES].

[20] E. Accomando et al., *Workshop on CP studies and non-standard Higgs physics*, hep-ph/0608079 [SPIRES].

JHEP04(2009)070

[21] C. Ruwiedel, N. Wermes and M. Schumacher, *Prospects for the measurement of the structure of the coupling of a Higgs boson to weak gauge bosons in weak boson fusion with the ATLAS detector*, Eur. Phys. J. **C 51** (2007) 385 [SPIRES].

[22] B.A. Dobrescu, G.L. Landsberg and K.T. Matchev, *Higgs boson decays to CP-odd scalars at the Tevatron and beyond*, Phys. Rev. **D 63** (2001) 075003 [hep-ph/0005308] [SPIRES].

[23] R. Dermisek and J.F. Gunion, *Escaping the large fine tuning and little hierarchy problems in the next to minimal supersymmetric model and $h \to aa$ decays*, Phys. Rev. Lett. **95** (2005) 041801 [hep-ph/0502105] [SPIRES].

[24] S. Chang, R. Dermisek, J.F. Gunion and N. Weiner, *Nonstandard Higgs boson decays*, Ann. Rev. Nucl. Part. Sci. **58** (2008) 75 [arXiv:0801.4554] [SPIRES].

[25] L. Álvarez-Gaumé and P.H. Ginsparg, *Geometry anomalies*, Nucl. Phys. **B 262** (1985) 439 [SPIRES].

[26] J. Preskill, *Gauge anomalies in an effective field theory*, Ann. Phys. **210** (1991) 323 [SPIRES].

[27] R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson*, Nucl. Phys. **B 671** (2003) 148 [hep-ph/0306259] [SPIRES].

[28] B. Gripaios, *Anomaly holography, the Wess-Zumino-Witten term and electroweak symmetry breaking*, Phys. Lett. **B 663** (2008) 419 [arXiv:0803.0497] [SPIRES].

[29] C.T. Hill and R.J. Hill, *Topological physics of little Higgs bosons*, Phys. Rev. **D 75** (2007) 115009 [hep-ph/0701044] [SPIRES].

[30] W. Kilian, D. Rainwater and J. Reuter, *Pseudo-axions in little Higgs models*, Phys. Rev. **D 71** (2005) 015008 [hep-ph/0411213] [SPIRES].

[31] C. Csáki, J. Heinonen, J. Hubisz and Y. Shirman, *Odd decays from even anomalies: gauge mediation signatures without SUSY*, arXiv:0901.2933 [SPIRES].

[32] E. Witten, *Global aspects of current algebra*, Nucl. Phys. **B 223** (1983) 422 [SPIRES].

[33] J. McDonald, *Gauge singlet scalars as cold dark matter*, Phys. Rev. **D 50** (1994) 3637 [hep-ph/0702143] [SPIRES].

[34] J. McDonald, *Electroweak baryogenesis and dark matter via a gauge singlet scalar*, Phys. Lett. **B 323** (1994) 339 [SPIRES].

[35] H. Davoudiasl, R. Kitano, T. Li and H. Murayama, *The new minimal standard model*, Phys. Lett. **B 609** (2005) 117 [hep-ph/0405097] [SPIRES].

[36] G.W. Anderson and L.J. Hall, *The electroweak phase transition and baryogenesis*, Phys. Rev. **D 45** (1992) 2685 [SPIRES].